Comment on the scaling function in $AdS_4 \times \mathbb{CP}^3$

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ABSTRACT: The folded spinning string in AdS_3 gives us an important insight into AdS/CFT duality. Recently its one-loop energy was analyzed in the context of AdS_4/CFT_3 by McLoughlin and Roiban arXiv:0807.3965, by Alday, Arutyunov and Bykov arXiv:0807.4400 and by Krishnan arXiv:0807.4561. They computed the spectrum of the fluctuations around the classical solution.

In this paper we reproduce their results using the algebraic curve technique and show that under some natural resummation of the fluctuation energies the one-loop energy agrees perfectly with the predictions of arXiv:0807.0777. This provides a further support of the all-loop Bethe equations and of the $AdS_4 \times \mathbb{CP}^3$ algebraic curve developed in arXiv:0807.0437.

Contents

1 Introduction

Integrability in AdS/CFT duality [1] is an exciting subject in the modern theoretical physics. The integrability of 4D Yang-Mills theory was first discussed in [2, 3]. An intensive development started after the seminal paper by Minahan and Zarembo [4] where the long single trace operators of $\mathcal{N} = 4$ Super Yang-Mills theory, dual to the superstring in the $AdS_5 \times S^5$ background, were mapped onto integrable spin chains. The string theory was shown to be classically integrable [5] and is widely believed to be integrable at the quantum level as well. Many attempts were made towards complete understanding of the spectrum of this system. In particular, the asymptotic Bethe ansatz proposed in [6–12] was an important step in this direction.

Recently, a new duality was discovered between $\mathcal{N} = 6$ super Chern-Simons theory with $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge group at level k and superstring theory in the $AdS_4 \times \mathbb{CP}^3$ background in the large N limit with the 't Hooft coupling $\lambda = N/k$ kept fixed [13, 14]. Amazingly, the gauge side of the duality also exhibits integrability [15] (see also [16, 17]). The string theory turns out to be classically integrable as well [18, 19]. The asymptotic all-loop Bethe equations were proposed recently in [20] and have already passed several independent tests [21, 22].

An important consequence of the classical integrability is an existence of the finite gap algebraic curve [23, 24]. For the $AdS_4 \times \mathbb{CP}^3$ superstring theory it was constructed in [25]. The algebraic curve describes all classical solutions in a unified gauge-invariant way and can be also used for computation of the spectrum of quantum fluctuations around a given classical solution [26]. We will use this technique to compute fluctuation frequencies around a particularly important folded string classical solution. These frequencies were obtained recently in [27–29] by a direct diagonalization of the string action expanded around the classical solution. The match of our results provides a nontrivial test of the algebraic curve construction [25].

The folded string is rotating in AdS_3 with large spin S and with angular momentum $J \sim \log S$ in $\mathbb{CP}^3$. The difference between its energy and spin scales as J [30, 31] for all

values of the 't Hooft coupling λ [13]. A similar solution in the context of AdS_5/CFT_4 duality has been already well studied for all values of λ [32–50]. In [20] an equation describing the $sl(2)$ sector of the $AdS_4 \times \mathbb{CP}^3$ string was proposed:

$$\left(\frac{x_k^+}{x_k^-}\right)^J = -\prod_{j \neq k}^{S} \left(\frac{x_k^+ - x_j^-}{x_k^- - x_j^+}\right)^{-1} \frac{1 - 1/(x_k^+ x_j^-)}{1 - 1/(x_k^- x_j^+)} \sigma_{\text{BES}}^2(u_k, u_j) . \tag{1.1}$$

It describes, in particular, the folded string. One can see that the only difference with $AdS_5 \times S^5$ string Bethe ansatz [51] is a minus sign on the r.h.s. It produces a simple redefinition of a scaling parameter

$$\ell_{AdS_5} = \frac{J}{4g \log S} \quad \rightarrow \quad \ell_{AdS_4} = \frac{J}{2h \log S} , \tag{1.2}$$

where $g = \sqrt{\lambda_{\text{YM}}}/4\pi$ and $h(\lambda_{\text{CS}}) = \sqrt{\lambda_{\text{CS}}/2} + \mathcal{O}(1/\sqrt{\lambda_{\text{CS}}})$ for large λ_{CS} and $h(\lambda_{\text{CS}}) = \lambda_{\text{CS}} + \mathcal{O}(\lambda_{\text{CS}}^3)$ for small λ_{CS} [16, 52–54]. This implies, in turn, that once the energy of the string in $AdS_5 \times S^5$ is given by

$$\gamma_{\text{YM}} = J \frac{f(g, \ell_{AdS_5})}{\ell_{AdS_5}} , \tag{1.3}$$

the energy of the folded string in $AdS_4 \times \mathbb{CP}^3$ reads [20]

$$\gamma_{\text{CS}} = J \frac{f(h, \ell_{AdS_4})}{\ell_{AdS_4}} , \tag{1.4}$$

where the one-loop scaling function f is given by [55]

$$f(h, \ell) \simeq \left(\sqrt{\ell^2 + 1} - \ell\right) + \frac{\sqrt{\ell^2 + 1} - 1 + 2(\ell^2 + 1) \log\left(1 + \frac{1}{\ell^2}\right) - (\ell^2 + 2) \log \frac{\sqrt{\ell^2 + 2}}{\sqrt{\ell^2 + 1} - 1}}{4\pi h \sqrt{\ell^2 + 1}} . \tag{1.5}$$

In the limit $\ell \to 1$ one finds

$$f(h, 0) \simeq 1 - \frac{3 \log 2}{4\pi h} . \tag{1.6}$$

Recently the folded string solution at one-loop was analyzed in $AdS_4 \times \mathbb{CP}^3$ [27–29] starting from a superstring action. In these papers the spectrum of fluctuations around the folded string was computed and summed up into a one-loop shift. A disagreement with eqs. (1.4), (1.5) was stated. According to these works, in the limit $\ell \to 0$ the scaling function is equal to $-\frac{5 \log 2}{4\pi h}$.

In this paper we reproduce the results for the fluctuation energies from the finite gap algebraic curve and propose a natural regularization of the sum of fluctuations leading precisely to the conjectured result (1.4), (1.5).

2 Algebraic curve for the folded string

In this section we recall the method of finding the fluctuation energies directly from the algebraic curve [26, 56, 57]. The algebraic curve allows us to compute these fluctuation energies disregarding a particular parametrization and gauge fixing of the superstring action.
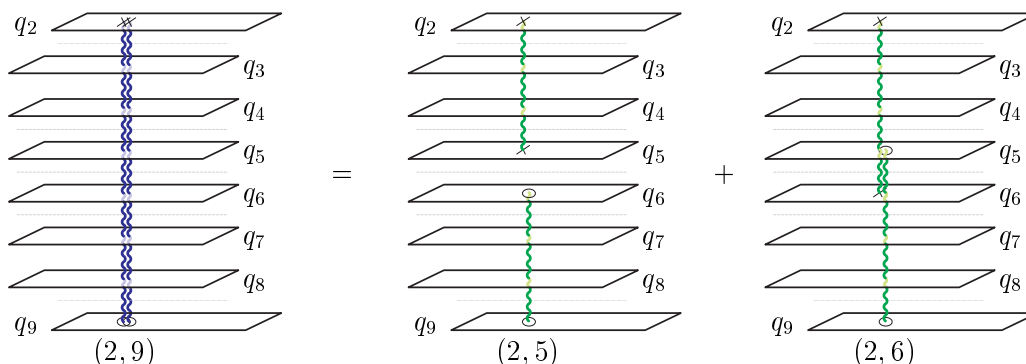
Figure 1. Equation relating fermionic and bosonic fluctuations (2,5) and (2,9). Adding a pole with double residue between sheets 2 and 9 is equivalent to adding a (2,5) pole plus a (2,6) pole. The fermionic fluctuations with polarizations (2,5) and (2,6) are equivalent since $q_5 = q_6$.

This provides us with a universal framework for computation of the one-loop corrections to the classical energies for superstrings living in different integrable backgrounds. The finite-gap solutions with any number of cuts can be treated on equal grounds [58–60]. Moreover, the calculations are much simpler then by the other methods. In particular, for the folded string solutions all the fluctuations can be obtained just from comparing the algebraic curves for the $AdS_5 \times S^5$ and $AdS_4 \times \mathbb{CP}^3$ superstrings.

We will start with describing the properties of the algebraic curves. The quasimomenta for the classical solution of string sigma-model in $AdS_4 \times \mathbb{CP}^3$ are not independent, namely,

$$\{q_1, \ q_2, \ q_3, \ q_4, \ q_5\} = -\{q_{10}, \ q_9, \ q_8, \ q_7, \ q_6\}. \tag{2.1}$$

Here the quasimomenta q_1, q_2, q_9, q_{10} are responsible for the AdS_4 part of sigma-model and the others for the $\mathbb{CP}^3$ part [25].

Since the motion of the folded string is constrained to $AdS_3 \times S^1$ subspace, the AdS-quasimomenta q_1, q_2 coincide with those for the folded string in $AdS_5 \times S^5$. They have two cuts shared with the functions q_{10} and q_9, respectively. The quasimomenta in $\mathbb{CP}^3$ are the same as in the point-like string limit (BMN limit), because the motion in this subspace is trivial,

$$q_3(x) = q_4(x) = -q_8(x) = -q_7(x) = \frac{2\pi\nu x}{x^2 - 1},$$
$$q_5(x) = -q_6(x) = 0. \tag{2.2}$$

Let us briefly recall the idea of quantization of the string using the algebraic curve. In order to obtain quasiclassical frequencies one has to perturb the classical curve and find the corresponding shift of the energy. These perturbations of the quasimomenta are implemented by adding some poles playing a role of infinitesimal cuts. We denote the polarization of the excitation by a pair of numbers (i,j) corresponding to the sheets between

which the pole is added. If one sheet corresponds to AdS and another to $\mathbb{CP}$, the fluctuation is fermionic, otherwise it is bosonic.

The residues of the poles are fixed by quasiclassical quantization condition and their positions are determined by the equations

$$q_i(x_n^{(i,j)}) - q_j(x_n^{(i,j)}) = 2\pi n_{(i,j)} \,. \tag{2.3}$$

Thus the calculation splits into two steps: (i) calculating the response of the energy $\delta E = \Omega_{ij}(x)$ to insertion of a pole at some point x and (ii) solving the equations 2.3. Then the fluctuation energy is simply $\delta E_n^{(i,j)} = \Omega_{ij}(x_n^{(i,j)})$.

In our case one can notice that all the functions $q_i(x)$ have already appeared in the $AdS_5 \times S^5$ algebraic curve, except for the trivial functions q_5 and q_6. This allows us to write immediately most of the fluctuation frequencies. For example, the fermionic excitation energy $\delta E_n^{(2,7)}$ is obtained by adding a pole between a sheet with two cuts in the physical region and a BMN-like sheet (i.e. containing only two poles at ± 1). In the $AdS_5 \times S^5$ such fluctuation has polarization $(\hat{2}, \tilde{3})$ and hence its frequency is equal to ω_n^F given in table 1. In the same way one finds the other $\delta E_n^{(i,j)}$'s, not involving the sheets 5 and 6, which we call "heavy" fluctuations.

For the fluctuations $\delta E_n^{(i,j)}$ with $j = 5$ or 6, to which we refer as "light", one can also avoid explicit calculations using a simple trick illustrated in figure 1 and figure 2. Let us for example consider figure 1. The prescription of [25] says that for the (2,9) and (1,10) fluctuations the residue should be doubled compared to other excitations. By that reason we depict them by a double line. This configuration can be thought of as adding a (2,5) pole (and automatically a "mirror" (6,9) pole, due to (2.1)), plus a pole between the second and the sixth sheets (and hence also a (5,9) pole). The poles on the intermediate sheets have opposite residues and cancel. Both configurations on the right hand side of figure 1 are equivalent and correspond to the fermionic fluctuation with polarization (2,5). Hence we find that $\Omega_{25}(x) = \frac{1}{2}\Omega_{29}(x)$.

At the next stage one has to solve the equation for the positions of the poles. From (2.1) and (2.2) it is clear that the solutions for the positions of the polarizations (2,5) and (2,9) are related by

$$x_n^{25} = x_{2n}^{29} \,. \tag{2.4}$$

Collecting all together we conclude that $\omega_n^{(2,5)} = \frac{1}{2}\omega_{2n}^{(2,9)}$. The similar trick gives the remaining frequencies $\omega_n^{(1,10)}$ and $\omega_n^{(3,7)}$ as one can see from the figure 2.

3 Summation prescription

Having the fluctuation frequencies $\omega_n^{(i,j)}$ computed we are ready to compute the shift of the classical value of the energy due to zero point oscillations. Formally one can write

$$\delta E_{1-loop} = \frac{1}{2} \sum_{(i,j)} (-1)^{F_{(i,j)}} \sum_n \omega_n^{(i,j)} \,, \tag{3.1}$$

however each sum over n for a given polarization (i,j) is divergent. A naive regularization which indeed leads to a finite result is to interchange the order of summation over n and (i,j). Then the cancelations between fermions and bosons make the sum over n convergent.
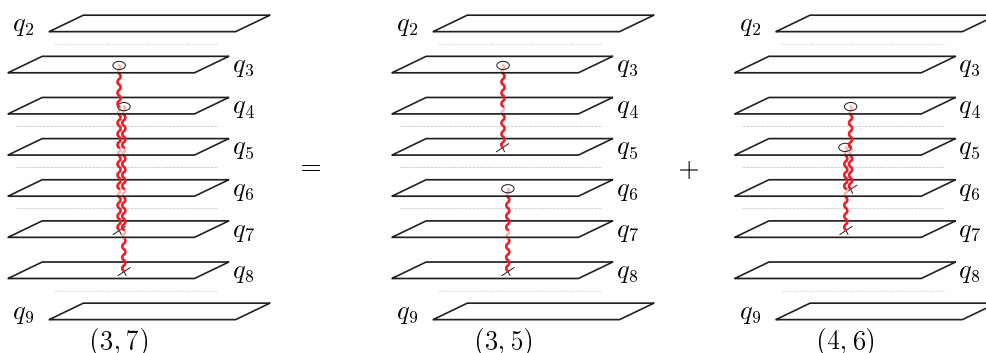
Figure 2. Equation relating fluctuations inside $\mathbb{CP}^3$ with polarizations (3,7) and (3,5). The (3,7) fluctuation can be decomposed into a (3,5) fluctuation plus (4,7) fluctuation. The fluctuations (3,5) and (4,7) for the folded string are obviously equivalent.

eigenmodes	notation
$\sqrt{n^2 + 2\kappa^2 \pm 2\sqrt{\kappa^4 + n^2\nu^2}}$	$\omega_n^{A\pm}$
$\sqrt{n^2 + 2\kappa^2 - \nu^2}$	ω_n^A
$\sqrt{n^2 + \kappa^2}$	ω_n^F
$\sqrt{n^2 + \nu^2}$	ω_n^S

Table 1. Notations for the frequencies of the folded string for $J \sim \log S \to \infty$. The frequencies are taken from [55]. In the table we use notations $\nu = \frac{J}{2\pi h}$, $\frac{\nu}{\kappa} = \frac{\ell}{\sqrt{\ell^2 + 1}}$.

	frequency	multiplicity	polarizations
AdS	ω_n^{A+}	$\times 1$	$(1, 10)$
	ω_n^{A-}	$\times 1$	$(2, 9)$
	ω_n^A	$\times 1$	$(1, 9)$
fermions	ω_n^F	$\times 4$	$(1, 7); (1, 8); (2, 7); (2, 8)$
	$\omega_{2n}^{A+}/2$	$\times 2$	$(1, 5); (1, 6)$
	$\omega_{2n}^{A-}/2$	$\times 2$	$(2, 5); (2, 6)$
$\mathbb{CP}^3$	ω_n^S	$\times 1$	$(3,7)$
	$\omega_{2n}^S/2$	$\times 4$	$(3, 5); (3, 6); (4, 5); (4, 6)$

Table 2. $3 + 8 + 5$ fluctuations around the folded string solution with multiplicities. Polarizations indicate which pair of sheets should be perturbed by a tiny pole.

In the context of $AdS_5 \times S^5$ this regularization leads to the agreement with the Bethe

ansatz prediction. In $AdS_4 \times \mathbb{CP}^3$ case several groups [27–29] stated a disagreement with the prediction from the all-loop Bethe ansatz [20].[1] Unfortunately in both cases a solid justification of the prescription used is missing. We present another regularization which leads to a different finite result. We argue by different means that our regularization is indeed the one which should be used for $AdS_4 \times \mathbb{CP}^3$.

The main difference between $AdS_4 \times \mathbb{CP}^3$ and $AdS_5 \times S^5$ theories is existence of two dispersion relations in the BMN spectrum. Accordingly the BMN spectrum [16, 52, 53] of $AdS_4 \times \mathbb{CP}^3$ is naturally divided into two groups. Because of the differences in the diameters of AdS_4 and $\mathbb{CP}^3$ there are four fermions, three AdS fluctuations and one $\mathbb{CP}^3$ fluctuation which have the energies

$$\mathcal{E}_n = \frac{1}{\kappa}\sqrt{n^2 + \kappa^2} \, , \tag{3.2}$$

whereas the other four $\mathbb{CP}^3$ and four fermionic BMN fluctuations have the following spectrum

$$\epsilon_n = \frac{1}{2\kappa}\sqrt{4n^2 + \kappa^2} \, . \tag{3.3}$$

In the Bethe ansatz language the first group of the fluctuations has both momentum carrying roots u_4 and $u_{\bar{4}}$ excited, while the second group has only one of them. We call them heavy and light excitations respectively. For even n one can think about the fluctuations from the first group to be some kind of bound state of two "light" excitations (with zero binding energy):

$$\mathcal{E}_n = \epsilon_{n/2} + \epsilon_{n/2} \, . \tag{3.4}$$

Indeed, from this argument we see that for even n "heavy" fluctuations with mode number n and the "light" fluctuations with mode number $n/2$ belong to the same family and should be treated together. Looking at the fluctuations around the folded string table 2 we also see that they are clearly separated into two groups.[2] We define

$$K_n = \begin{cases} \omega_n^{\text{heavy}} + \omega_{n/2}^{\text{light}} & n \in \text{even} \\ \omega_n^{\text{heavy}} & n \in \text{odd} \end{cases} \tag{3.5}$$

where ω_n^{heavy} and ω_n^{light} in general are defined by

$$\omega_n^{\text{heavy}} = \omega_n^{(1,10)} + \omega_n^{(2,9)} + \omega_n^{(1,9)} - \omega_n^{(1,7)} - \omega_n^{(1,8)} - \omega_n^{(2,7)} - \omega_n^{(2,8)} + \omega_n^{(3,7)} \tag{3.6}$$

$$\omega_n^{\text{light}} = \omega_n^{(3,5)} + \omega_n^{(3,6)} + \omega_n^{(4,5)} + \omega_n^{(4,6)} - \omega_n^{(1,5)} - \omega_n^{(1,6)} - \omega_n^{(2,5)} - \omega_n^{(2,6)} \, .$$

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[1]More accurate analysis shows that the conclusion about disagreement with Bethe ansatz given in [27–29] is wrong and their results do agree with the asymptotical Bethe ansatz once the unknown interpolation function $h(\lambda)$ is fixed appropriately by adding a constant term λ^0 [61]. In this section we propose another resolution of the problem keeping the constant term to be zero.

In [61] also a circular string solution was considered. Only a partial agreement with Bethe ansatz was found in that case. We will explain in our forthcoming work with P.Vieira that the mismatch disappears once the coset string action [18] is used.

[2]Formally one can pass to the BMN limit $\nu = \kappa$ to distinguish them.

We claim that the one-loop shift of the string energy is given by

$$E_{1-\text{loop}} = \lim_{N \to \infty} \sum_{n=-N}^{N} \frac{K_n}{2\kappa} \ . \tag{3.7}$$

Using the notations given in table 2 one can pass to the fluctuations listed in table1. For example $\omega_n^{(1,10)} = \omega_n^{A+}$ and $\omega_n^{(3,5)} = \omega_{2n}^S/2$. From eq. (3.6) we get

$$\omega_n^{\text{heavy}} = \omega_n^{A+} + \omega_n^{A-} + \omega_n^A - 4\omega_n^F + \omega_n^S \tag{3.8}$$
$$\omega_{n/2}^{\text{light}} = 2\omega_n^S - \omega_n^{A+} - \omega_n^{A-} \ .$$

For large κ we can replace the sum with an integral

$$E_{1-\text{loop}} \simeq \lim_{N \to \infty} \int_{-N}^{N} \frac{2\omega_n^{\text{heavy}} + \omega_n^{\text{light}}}{4\kappa} dn = \int_0^\infty \left(2\omega_n^A + \omega_n^{A+} + \omega_n^{A-} + 4\omega_n^S - 8\omega_n^F \right) \frac{dn}{2\kappa} \ . \tag{3.9}$$

We notice that exactly this integral was computed in [55]. We immediately write the result

$$E_{1-\text{loop}} \simeq \frac{J}{\ell} \frac{\sqrt{\ell^2 + 1} - 1 + 2(\ell^2 + 1) \log\left(1 + \frac{1}{\ell^2}\right) - (\ell^2 + 2) \log \frac{\sqrt{\ell^2 + 2}}{\sqrt{\ell^2 + 1} - 1}}{4\pi h \sqrt{\ell^2 + 1}} \tag{3.10}$$

where we used that $\frac{\nu}{\kappa} = \frac{\ell}{\sqrt{\ell^2+1}}$ and $\nu = \frac{J}{2\pi h}$. eq. (3.10) agrees completely with eqs. (1.4), (1.5)! In particular, when $\ell \to 0$ we get

$$E_{1-\text{loop}} \simeq -\frac{3 \log 2}{2\pi} \log S \ . \tag{3.11}$$

4 Summary

In this paper we propose a particular regularization of the sum over the fluctuation frequencies. The regularization goes as follows: we split the fluctuations into two groups with 4 bosons and 4 fermions each. We call them heavy and light fluctuations since for even mode numbers n heavy ones can be decomposed into a sum of two light fluctuations with mode number $n/2$. The light fluctuations are in some sense more fundamental. We prescribe then the mode number $2n$ heavy fluctuations to be treated together with mode number n light fluctuations. Following this prescription we match the one-loop energy shift with the prediction from the all-loop Bethe equations. This regularization method looks a bit artificial on the n plane, but from the algebraic curve point of view it makes perfect sense. The positions of the excitations on the algebraic curve of the light and heavy excitations are related in the following way

$$x_{2n}^{\text{heavy}} = x_n^{\text{light}} \tag{4.1}$$

(see for example eq. (2.4)). We can regularize the sum over the fluctuations in terms of the algebraic curve by introducing an ϵ balls around $x = \pm 1$ and taking into account only the fluctuations living in their exterior. For the $AdS_5 \times S^5$ string this ϵ regularization leads to the simple cut-off prescription in the sum over fluctuations. However, in the $AdS_4 \times \mathbb{CP}^3$ case one should be more careful.

Whereas it is clear from the algebraic curve why this particular regularization makes sense it is rather hard to justify it starting from the world-sheet string action. At the end of the day the algebraic curve and the action are two equivalent descriptions of the semiclassical strings and it should be possible to understand this regularization from both points of view.[3] One of the possible approaches to better understanding of the problem is to pass to the Frolov-Tseytlin limit ($J \to \infty$ and J/h large) where the conjectured asymptotic all-loop Bethe equations reduce to the two-loop Bethe equations derived from the CS perturbation theory [15]. Our hope is that the finite-size corrections to the scaling limit of the two-loop Bethe equations will only be consistent with the sum over fluctuations if our prescription is used.

Acknowledgments

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[3]It is entirely possible that classically equivalent theories become inequivalent at the quantum level. However it is widely believed that once all relevant symmetries of the system are conserved the quantization is unique. On the algebraic curve all the symmetries are explicit and thus it seems to be the best starting point for the quantization.

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